FJ is trying to qualify for the school team in the shot put event, which involves throwing a heavy metal ball called a shot. In order to qualify, the shot must land on the ground at least 48 feet from where FJ is standing. For FJ's farthest throw, the shot is thrown at an angle of 36.87° with the horizontal, at an initial speed of 40 feet per second, from an initial height of 6 feet. **SCORE: _____ / 25 PTS**

NOTE: $\cos 36.87^\circ = \frac{4}{5}$ and $\sin 36.87^\circ = \frac{3}{5}$

[a] Write parametric equations for the position of the shot.

10

$$x = (v_0 \cos\theta)t = 40 \cdot \tfrac{4}{5}t = 32t \quad 2$$

$$y = h_0 + (v_0 \sin\theta)t - 16t^2 = 6 + 40 \cdot \tfrac{3}{5}t - 16t^2 = 6 + 24t - 16t^2$$

2 2 2 2

[b] Does FJ qualify for the team ?

15

$48 = 32t \rightarrow t = \frac{3}{2}$ 2

3 $y = 6 + 24\left(\frac{3}{2}\right) - 16\left(\frac{3}{2}\right)^2$ 3

$= 6 + 36 - 36$

3

2 $= 6 > 0 \longrightarrow$ SHOT WAS STILL IN THE AIR AT 48 FT MARK

2

FJ QUALIFIED FOR THE TEAM

Using mathematical induction, prove that $\sum_{i=1}^{n} [i(3i-1)] = n^2(n+1)$ for all positive integers n. **SCORE: _____ / 25 PTS**

1 1 1 1

BASIS STEP: $\sum_{i=1}^{1}(i(3i-1)) = 1 \cdot (3-1) = 2 = 1^2(1+1)$

2

INDUCTIVE STEP: ASSUME $\sum_{i=1}^{k}(i(3i-1)) = k^2(k+1)$ FOR SOME PARTICULAR BUT ARBITRARY INTEGER $k \geq 1$

2

2 $\sum_{i=1}^{k+1}(i(3i-1)) = \sum_{i=1}^{k}(i(3i-1)) + (k+1)(3(k+1)-1)$ 3

2 $= k^2(k+1) + (k+1)(3k+2)$ 2

$= (k+1)(k^2+3k+2)$

$= (k+1)(k+1)(k+2)$ ALGEBRAIC PROOF 5

$= (k+1)^2((k+1)+1)$ 2

BY MI, $\sum_{i=1}^{n}(i(3i-1)) = n^2(n+1)$ FOR ALL INTEGERS $n \geq 1$ 1

Find the sum of the series $125+117+109+101+\cdots-235$. **SCORE: _____ / 15 PTS**

ARITHMETIC , $d=-8$

$\underline{-235=125+(-8)(n-1)}$ 4

$-360=-8(n-1)$

$45=n-1$

$\underline{n=46}$ 2

$S_{46}=\boxed{\frac{46}{2}}(\overbrace{125}^{2}+\overbrace{^{-}235}^{2})$ 2

$=23(-110)$

$=\underline{-2530}$ 3

Consider the expression $(13x^5-11x^2)^{47}$. **SCORE: _____ / 30 PTS**

[a] Write the expansion of the expression using sigma notation. Your answer may use ${}_nC_r$ notation, × and positive exponents only. Simplify all exponents.

12

$$\sum_{r=0}^{47} \underbrace{{}_{47}C_r}_{1} \underbrace{(13x^5)^{47-r}}_{} \underbrace{(-11x^2)^r}_{1}$$

$$=\sum_{r=0}^{47} {}_{47}C_r\, 13^{47-r}(-11)^r x^{235-5r} x^{2r}$$

$$=\underbrace{\boxed{\sum_{r=0}^{47}}}_{2}\underbrace{(-1)^r}_{1}\underbrace{{}_{47}C_r}_{2}\underbrace{13^{47-r}}_{1}\underbrace{11^r}_{1}\underbrace{x^{235-3r}}_{3}$$

-2 IF INDEX DOESN'T MATCH INSIDE OF Σ

[b] Find the 38^{th} term in the expansion. Your answer may use ${}_nC_r$ notation, +, –, × and positive exponents only.

6

$\underline{r=37}$

1 EACH

$(-1)^{37}{}_{47}C_{37}\,13^{47-37}\,11^{37}\,x^{235-3(37)} = \underline{-}\,\underline{{}_{47}C_{37}}\,\underline{13^{10}}\,\underline{11^{37}}\,\underline{x^{124}}$

[c] Find the coefficient of x^{187} in the expansion. Your answer may use ${}_nC_r$ notation, +, –, × and positive exponents only.

12

$\underline{235-3r=187}$ 6

$-3r=-48$

$\underline{r=16}$ 2

1 NO NEGATIVE

$$(-1)^{16}{}_{47}C_{16}\,13^{47-16}\,11^{16} = \underbrace{{}_{47}C_{16}}_{1}\,\underbrace{13^{31}}_{1}\,\underbrace{11^{16}}_{1}$$

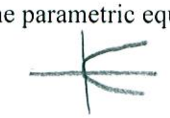
GJ was taught to save money starting at a very young age. On her 12^{th} birthday, GJ opened a savings account and deposited $\$240$ into it. Every birthday afterwards, GJ made another deposit which was 4% more than the deposit made the previous birthday. After the deposit on her 21^{st} birthday, what was the total amount that had been deposited in the account ? **SCORE: _____ / 15 PTS**

$240 + 240(1.04) + 240(1.04)^2 + \cdots + 240(1.04)^9$ 5

$= \dfrac{240(1.04^{10}-1)}{1.04-1}$ 5 5 $= \dfrac{240(1.04^{10}-1)}{0.04} = \$2881.47$

Describe the difference between the curves with parametric equations 1 $\begin{array}{l} x = \cos^2 t \\ y = \cos t \end{array}$ and $\begin{array}{l} x = t^6 \\ y = t^3 \end{array}$ 2. **SCORE: _____ / 15 PTS**

Discuss the rectangular equation(s) of the graphs, as well as the orientation and portion of the graph corresponding to the parametric equations.

BOTH CURVES ARE PART OF THE PARABOLA $x = y^2$ 5 

AS t GOES FROM $-\infty$ TO ∞

1 $y = \cos t$ OSCILLATES BETWEEN -1 AND 1

5 SO CURVE GOES BACK + FORTH BETWEEN $(1,-1)$ AND $(1,1)$

2 $y = t^3$ GOES FROM $-\infty$ TO ∞

5 SO CURVE GOES FROM BOTTOM RIGHT TO TOP RIGHT

Use sigma notation to write the series $\dfrac{\overset{2!}{2}}{486} - \dfrac{\overset{3!}{6}}{324} + \dfrac{\overset{4!}{24}}{216} - \dfrac{\overset{5!}{120}}{144} + \cdots - \dfrac{\overset{7!}{5040}}{64}$. ← GEOMETRIC, $r = \frac{2}{3}$ **SCORE: _____ / 10 PTS**

$\displaystyle\sum_{n=1}^{6} (-1)^{n+1} \frac{(n+1)!}{486\left(\frac{2}{3}\right)^{n-1}}$ 2 2 3 3 OR $\displaystyle\sum_{n=2}^{7} (-1)^{n} \frac{n!}{486\left(\frac{2}{3}\right)^{n-2}}$

OR $\displaystyle\sum_{n=0}^{5} (-1)^{n} \frac{(n+2)!}{486\left(\frac{2}{3}\right)^{n}}$

-2 IF INDEX DOESN'T MATCH INSIDE OF Σ

A tax rebate has been given to property owners by the state government with the anticipation that each property owner will spend approximately 85% of the rebate, and in turn each recipient of this amount will spend 85% of what they receive, and so on. Economists refer to this exchange of money and its circulation within the economy as the "multiplier effect". The multiplier effect operates on the idea that the expenditures of one individual become the income of another individual. For a tax rebate of $\$600$, find the total amount put back into the state's economy, if this effect continues without end. **SCORE: _____ / 15 PTS**

$600 + 600(0.85) + 600(0.85)^2 + \cdots$ 6

$= \dfrac{600}{1-0.85}$ 6 $= \dfrac{600}{0.15} = \$4{,}000$ 3